The evaluation of synchrotron radiation in SAD is done usingbased on "kinematical method":
Let $\boldsymbol{q}$ denote the orientation vector of the momentum of a particle:

$$
\begin{align*}
\boldsymbol{q} & =\left(\frac{p_{x}}{p}, \frac{p_{y}}{p}, \frac{p_{z}}{p}\right),  \tag{81}\\
p_{z} & =\sqrt{p^{2}-p_{x}^{2}-p_{y}^{2}} . \tag{82}
\end{align*}
$$

Suppose a particles traverses a section $(1,2)$ of an accelerator component, then the orientation changes from $\boldsymbol{q}_{1}$ to $\boldsymbol{q}_{2}$. The bending angle $\phi$ and the radius of curvature $\rho_{\mathrm{r}}$ are approximated, assuming a uniform bending, by:

$$
\begin{align*}
& \sin |\phi|=\left|\boldsymbol{q}_{2} \times \boldsymbol{q}_{1}\right|,  \tag{83}\\
& \rho_{\mathrm{r}}=\frac{\mathrm{L}_{12}-z_{2}+z_{1}}{|\phi|}, \tag{84}
\end{align*}
$$

where $L_{12}$ is the nominal length of the component between 1 and 2 , and $z_{1,2}$ are the values of longitudinal coordinate $z \equiv-v\left(t-t_{0}\right)$ at the locations 1 and 2 .


Figure 1: The kinematical method for synchrotron radiation.

By knowing $\phi$ and $\rho_{\mathrm{r}}$ as well as the momentum of the particle, we can derive all information about the emission of synchrotron radiation (if we can use a classical formula with uniform bending).

- Thus the synchrotron radiation can be handled by a single routine for any type of component, such as multipoles, solenoid, fringe field, even including electric field, without knowing the details of the field.
- A component is sliced so that $N_{\gamma} \lesssim 1$.
- Not only the radiation itself, its derivatives by phase space coordinates can be obtained kinematically using the transfer matrix. These derivatives are used to evaluate the damping and excitation matrices.
- In the region where the field is not uniform, such as the F1 region of a BEND, a special treatment for $\rho_{\mathrm{r}}$ is applied.
- This method may be applied for a spin motion if the longitudinal filed is taken care properly.

