The Lagrangean L defines the *canonical* momenta as

$$p_x = \frac{\partial L}{\partial x'} = \frac{mcx'}{p_0\sqrt{c^2t'^2 - x'^2 - y'^2 - (1+x/\rho)^2}} + a_x\,, \tag{6}$$

$$p_y = \frac{\partial L}{\partial y'} = \frac{mcy'}{p_0\sqrt{c^2t'^2 - x'^2 - y'^2 - (1+x/\rho)^2}} + a_y\,, \tag{7}$$

$$p_t = \frac{\partial L}{\partial t'} = -\frac{mc^3t'}{p_0\sqrt{c^2t'^2 - x'^2 - y'^2 - (1+x/\rho)^2}}\,, \tag{8}$$

which derives the Hamiltonian as

$$H_t = x'p_x + y'p_y + t'p_t - L \tag{9}$$

$$= -\left(\sqrt{-c^2m^2/p_0^2 + p_t^2/c^2 - (p_x - a_x)^2 + (p_y - a_y)^2} + a_s\right)\left(1 + \frac{x}{\rho}\right)\,. \tag{10}$$

Instead of the canonical variables (t, p_t), SAD uses another set (z, p), The variable z means the logitudinal postion, and p the total momentum, which is more convenient than p_t especially in a low-energy case, ie., $\gamma \sim 1$. The canonical variables (z, p) as well as the Hamiltonian H are obtained using a mother function

$$G = G(p_t, z) = \frac{z}{c}\sqrt{p_t^2 - m^2c^4/p_0^2} - t_0(s)\,, \tag{11}$$

$$p = \frac{\partial G}{\partial z} = \frac{\sqrt{p_t^2p_0^2 - m^2c^4}}{p_0}\,, \tag{12}$$

$$t = \frac{\partial G}{\partial p_t} = -z\frac{\sqrt{p^2p_0^2 - m^2c^2}}{cpp_0} + t_0(s)\,, \tag{13}$$

$$H = H_t - \frac{\partial G}{\partial s} \tag{14}$$

$$= -\left(\sqrt{p^2 - (p_x - a_x)^2 - (p_y - a_y)^2} + a_s\right)\left(1 + \frac{x}{\rho}\right) + \frac{E}{p_0v_0}\,, \tag{15}$$

where $t_0(s)$ is the *design arrival time* at location s, $E = \sqrt{m^2c^4 + p_0^2p^2}$ the energy of the particle, and $v_0 = 1/t_0'(s)$ the design velocity. The longitudinal position z is written as

$$z = -v\left(t - t_0(s)\right)\,, \tag{16}$$

where v is the total velocity of the particle. Note that $z > 0$ for the head of a bunch.

Thus the canonical variables in SAD are:

$$(x, p_x, y, p_y, z, \delta \equiv p - 1)\,. \tag{17}$$