The primary position variables are (x, y, s), where x and y are the displacements along the normal and binormal vectors,  $\boldsymbol{n}$  and  $\boldsymbol{b}$ , respectively. Let  $\boldsymbol{t}$  denote the tangential vector along s, then  $\boldsymbol{n}$ ,  $\boldsymbol{b}$ ,  $\boldsymbol{t}$  consist a right-handed system.

The action in t is expressed by

$$S = \int L_t c dt \,, \tag{2}$$

$$L_t = -\frac{mc}{p_0}\sqrt{1 - \dot{x}^2 + \dot{y}^2 + (1 + x/\rho)^2 \dot{s}^2} + a_x \dot{x} + a_y \dot{y} + (1 + x/\rho)a_s \dot{s}, \qquad (3)$$

where  $p_0$  and  $(a_x, a_y, a_z) = e(A_x, A_y, A_z)/p_0$  are the design momentum and the normalized vector potentials, respectively, and  $\dot{}$  denotes the derivative by *ct*. SAD's coordinate only has the radius of curvature  $\rho$  in the local *x*-*s* plane. Note that  $\rho$  is the curvature of the coordinate system, not that of the orbit. The transverse vector potentials  $(a_x, a_y)$  are non-zero only in the solenoid region, where  $1/\rho$  is zero. Currently SAD does not handle the electrostatic potential.

As SAD uses s for the independent variable instead of t, the Lagrangean L for s is written as

$$L = L_t \frac{dct}{ds},$$

$$= -\frac{mc}{p_0} \sqrt{c^2 t'^2 - x'^2 + y'^2 + (1 + x/\rho)^2} + a_x x' + a_y y' + (1 + x/\rho) a_s,$$
(4)
(5)

where ' is the derivative by s.