The primary position variables are $(x, y, s)$, where $x$ and $y$ are the displacements along the normal and binormal vectors, $\boldsymbol{n}$ and $\boldsymbol{b}$, respectively. Let $\boldsymbol{t}$ denote the tangential vector along $s$, then $\boldsymbol{n}, \boldsymbol{b}, \boldsymbol{t}$ consist a right-handed system.
The action in $t$ is expressed by

$$
\begin{align*}
S & =\int L_{t} c d t  \tag{2}\\
L_{t} & =-\frac{m c}{p_{0}} \sqrt{1-\dot{x}^{2}+\dot{y}^{2}+(1+x / \rho)^{2} \dot{s}^{2}}+a_{x} \dot{x}+a_{y} \dot{y}+(1+x / \rho) a_{s} \dot{s} \tag{3}
\end{align*}
$$

where $p_{0}$ and $\left(a_{x}, a_{y}, a_{z}\right)=e\left(A_{x}, A_{y}, A_{z}\right) / p_{0}$ are the design momentum and the normalized vector potentials, respectively, and denotes the derivative by ct. SAD's coordinate only has the radius of curvature $\rho$ in the local $x$-s plane. Note that $\rho$ is the curvature of the coordinate system, not that of the orbit. The transverse vector potentials ( $a_{x}, a_{y}$ ) are non-zero only in the solenoid region, where $1 / \rho$ is zero.
Currently SAD does not handle the electrostatic potential.
As SAD uses $s$ for the independent variable instead of $t$, the Lagrangean $L$ for $s$ is written as

$$
\begin{align*}
L & =L_{t} \frac{d c t}{d s}  \tag{4}\\
& =-\frac{m c}{p_{0}} \sqrt{c^{2} t^{\prime 2}-x^{\prime 2}+y^{\prime 2}+(1+x / \rho)^{2}}+a_{x} x^{\prime}+a_{y} y^{\prime}+(1+x / \rho) a_{s}, \tag{5}
\end{align*}
$$

where ${ }^{\prime}$ is the derivative by $s$.

