

The transformation of a DECA is given as

$$\begin{aligned} & \exp(: F_{\text{in}} :) \exp(: aL :) \exp(: H_4/2 :) \exp(: bL :) \\ & \times \exp(: V_4 :) \exp(: aL :) \exp(: H_4/2 :) \exp(: bL :) \exp(: F_{\text{out}} :) , \end{aligned} \quad (106)$$

where L and H_4 are the Hamiltonians of a drift of length L and a thin decapole kick with integrated strength $K4$:

$$H_4 = \frac{K4}{5!} \Re(x - iy)^5 , \quad (107)$$

respectively. The coefficients are $a \equiv 1/2 - 1/\sqrt{12}$ and $b = 1/2 - a$. Terms $\exp(: F_{\text{in}} :)$ and $\exp(: F_{\text{out}} :)$ are transformations for entrance and exit nonlinear fringes. The term $\exp(: V_4 :)$ is a correction to adjust the third-order terms in L :

$$V_4 = \sum_{j=(x,y), k=(x,y)} -\frac{\beta}{2} H_{4,k}^2 + \gamma H_{4,j} H_{4,k} H_{4,j,k} , \quad (108)$$

where $,i$ represents the derivative by x or y . We have also introduced two coefficients $\beta \equiv 1/6 - 1/\sqrt{48}$ and $\gamma = 1/40 - 1/(24\sqrt{3})$.